1D mathe II – corrigé   Examen 2001-2002

1) manuel p. 25

2) $\lim\limits_{x\to 0} \dfrac{x \operatorname{Arcsin} x}{1-\cos x} = \dfrac{0}{0}$ (f.i.)

$\underset{(H)}{=} \lim\limits_{x\to 0} \dfrac{\operatorname{Arcsin} x + x\cdot \frac{1}{\sqrt{1-x^2}}}{\sin x} = \dfrac{0}{0}$ (f.i.)

$\underset{(H)}{=} \lim\limits_{x\to 0} \dfrac{\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} + x\cdot(-\frac{1}{2})(1-x^2)^{-\frac{3}{2}}\cdot(-2x)}{\cos x}$

$= \lim\limits_{x\to 0} \dfrac{\frac{2}{\sqrt{1-x^2}} + \frac{x^2}{\sqrt{(1-x^2)^3}}}{\cos x}$

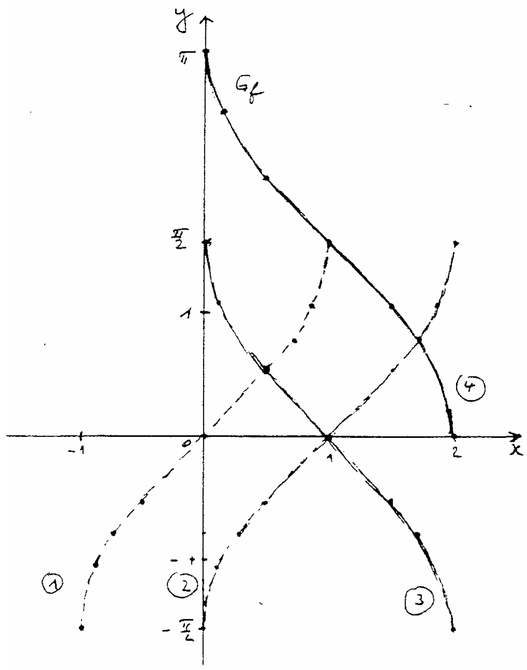
$= 2$

3) a) ① $y = \operatorname{Arcsin} x$

② $y = \operatorname{Arcsin}(x-1)$ — $t_{\vec{i}}$

③ $y = -\operatorname{Arcsin}(x-1)$ — symétrie par rapport à Ox

④ $y = \frac{\pi}{2} - \operatorname{Arcsin}(x-1)$ — $t_{\frac{\pi}{2}\cdot\vec{j}}$



b) $A = \int_0^2 \left[\frac{\pi}{2} - \operatorname{Arcsin}(x-1)\right] dx$

$= \frac{\pi}{2}[x]_0^2 - \int_0^2 \operatorname{Arcsin}(x-1)\,dx$

$t = x-1 \Leftrightarrow x = t+1$

$\frac{dt}{dx} = 1 \Rightarrow dt = dx$

x	0	2
t	−1	1

$= \frac{\pi}{2}(2-0) - \underbrace{\int_{-1}^{1} \operatorname{Arcsin} t\, dt}_{0 \text{ car}}$

Arcsinus est une fonction impaire

$= \pi$ u.a

$\boxed{A = 4\pi \text{ cm}^2}$

---

1) manuel p. 55

2) a) $\log_{0,5}(2x^2-x-1) \le 1 - \log_2(2x+4)$ (I)

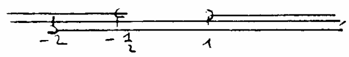
Cond : $2x^2-x-1>0$ et $2x+4>0$

$\Leftrightarrow (x<-\frac{1}{2}$ ou $x>1)$ et $x>-2$

donc $I =]-2, -\frac{1}{2}[\cup]1, +\infty[$

$\Delta = 1+8 = 9$

$x_{1,2} = \frac{1\pm 3}{4} = \begin{cases} -\frac{1}{2} \\ 1 \end{cases}$



$(I) \Leftrightarrow \log_{0,5}(2x^2-x-1) \le \log_{0,5}(0,5) + \log_{0,5}(2x+4)$

$\Leftrightarrow \log_{0,5}(2x^2-x-1) \le \log_{0,5}[0,5(2x+4)]$

$\Leftrightarrow 2x^2-x-1 \ge x+2$ et $x \in \text{dom } I$

$\Leftrightarrow 2x^2-2x-3 \ge 0$ et $x \in \text{dom } I$

$\Delta' = 1+6 = 7$

$x_{1,2} = \frac{1 \pm \sqrt{7}}{2} = \begin{cases} 1,82 \\ -0,82 \end{cases}$

$\Leftrightarrow x \le \frac{1-\sqrt{7}}{2}$ ou $x \ge \frac{1+\sqrt{7}}{2}$ et $x \in \text{dom } I$

$$\boxed{S_{\mathbb{R}} = \left]-2, \frac{1-\sqrt{7}}{2}\right] \cup \left[\frac{1+\sqrt{7}}{2}, +\infty\right[}$$

b) $\frac{e^x+3}{e^{x+1}-e} = e^{x-1}$ | $e(e^x-1)$

Cond : $e^{x+1}-e \neq 0 \Leftrightarrow e(e^x-1) \neq 0 \Leftrightarrow e^x \neq 1 \Leftrightarrow x \neq 0$

$\Leftrightarrow e^x+3 = e(e^x-1)e^{x-1}$ et $x \neq 0$

$\Leftrightarrow e^x+3 = (e^x-1)e^x$

$\Leftrightarrow e^x+3 = e^{2x}-e^x$

$\Leftrightarrow e^{2x}-2e^x-3 = 0$ $t^2-2t-3=0 \Leftrightarrow t = 1 \pm 2 = \begin{cases} 3 \\ -1 \end{cases}$

$\Leftrightarrow e^x = 3$ ou $\underbrace{e^x = -1}_{\text{impossible car } e^x > 0,\ \forall x \in \mathbb{R}}$

$\Leftrightarrow x = \ln 3$

$$\boxed{S_{\mathbb{R}} = \{\ln 3\}}$$

III. $f(x) = \frac{e^x}{\sqrt{e^x-1}}$

1) * $x \in \text{dom} f \Leftrightarrow e^x-1 > 0 \Leftrightarrow e^x > 1 \Leftrightarrow x > 0$

$\text{dom} f =]0,+\infty[= \text{dom}_c f$ (quotient de fonctions continues)

* $\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{e^x}{\sqrt{e^x-1}} = \frac{+\infty}{+\infty}$ F.I.

$= \lim_{x \to +\infty} \frac{e^x}{\sqrt{e^x} \cdot \sqrt{1-e^{-x}}}$

$= \lim_{x \to +\infty} \frac{\sqrt{e^x}}{\sqrt{1-e^{-x}}} = \frac{+\infty}{1} = +\infty$ (pas d'AH)

$\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \underbrace{\frac{\sqrt{e^x}}{x}}_{\to +\infty} \cdot \underbrace{\frac{1}{\sqrt{1-e^{-x}}}}_{\to 1}$ (l'exp. l'emporte sur la puiss.)

$= +\infty$ (pas d'A.O. ; BP de direction Oy)

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{e^x}{\sqrt{e^x-1}} = +\infty \ \left(\frac{1}{0^+}\right) \Rightarrow$ A.V. d'éq. $x = 0$

$]\dots, +\infty[, \quad f'(x) = \dfrac{\sqrt{e^x-1}\cdot e^x - e^x \cdot \frac{e^x}{2\sqrt{e^x-1}}}{e^x-1}$

$= \dfrac{2(e^x-1)\cdot e^x - e^{2x}}{2(e^x-1)\sqrt{e^x-1}}$

$= \dfrac{2e^{2x}-2e^x-e^{2x}}{2(e^x-1)\sqrt{e^x-1}}$

$= \dfrac{e^{2x}-2e^x}{2(e^x-1)\sqrt{e^x-1}}$

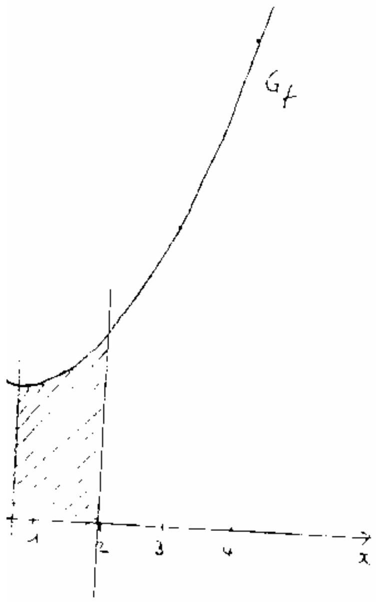
$$f'(x) = \frac{e^x(e^x-2)}{2(e^x-1)\sqrt{e^x-1}}$$

$f'(x)=0 \Leftrightarrow e^x-2=0 \Leftrightarrow x=\ln 2$

$f'(x)>0 \Leftrightarrow e^x-2>0 \Leftrightarrow x>\ln 2$

$f(\ln 2) = 2$

x	0		$\ln 2$		$+\infty$
$f'(x)$		$-$	0	$+$	
$f(x)$	$+\infty$	$\searrow$	2 min. absolu	$\nearrow$	$+\infty$



x	0,25	0,5	$\ln 2$	1	2	3	[illegible]
y	2,41	2,05	2	2,07	2,9	4,60	7,

c) $\mathcal{A} = 2\displaystyle\int_{\ln 2}^{2} \frac{e^x}{2\sqrt{e^x-1}}\,dx \qquad \begin{cases} u(x) = e^x-1 \\ u'(x) = e^x \end{cases}$

$= 2\left[\sqrt{e^x-1}\right]_{\ln 2}^{2}$

$= 2\left(\sqrt{e^2-1} - 1\right)$

$= (2\sqrt{e^2-1} - 2)$ u.a

$\mathcal{A} \approx 3,06$ u.a.

$I = \displaystyle\int_{-1}^{2} \frac{x}{\sqrt{2x+3}}\,dx \qquad ([-1,2] \subset \operatorname{dom} f =]-\tfrac{3}{2}, +\infty[)$

$t = 2x+3 \Rightarrow x = \frac{t-3}{2}$

$\frac{dt}{dx} = 2 \Rightarrow dx = \frac{1}{2}dt$

x	-1	2
t	1	7

$= \displaystyle\int_1^7 \frac{\frac{t-3}{2}}{\sqrt{t}} \cdot \frac{1}{2}\,dt$

$= \frac{1}{4}\displaystyle\int_1^7 \frac{t-3}{\sqrt{t}}\,dt$

$= \frac{1}{4}\displaystyle\int_1^7 \left(\sqrt{t} - \frac{3}{\sqrt{t}}\right)dt$

$= \frac{1}{4}\left\{\frac{2}{3}[t\sqrt{t}]_1^7 - \dots[\sqrt{t}]_1^7\right\}$

$= \frac{1}{6}(7\sqrt{7}-1) - \frac{3}{2}(\sqrt{7}-1)$

$= (\frac{7}{6} - \frac{3}{2})\sqrt{7} - \frac{1}{6} + \frac{3}{2}$

$= \frac{4}{3} - \frac{\sqrt{7}}{3}$

$= \frac{4-\sqrt{7}}{3}$

$f(x) = 1 - \ln x^2$

a) $\text{dom} f = \mathbb{R}^*$

$\forall x \in \mathbb{R}^*, \quad f(x) = 1 - 2\ln|x|$

$f(x) \geq 0 \iff \ln|x| \leq \frac{1}{2}$

$\iff |x| \leq e^{\frac{1}{2}}$ et $x \neq 0$

$\iff \underline{-\sqrt{e} \leq x < 0 \text{ ou } 0 < x \leq \sqrt{e}}$

b) Sur $[1, \sqrt{e}]$, $f(x) \geq 0$

Sur $[\sqrt{e}, e]$, $f(x) \leq 0$

Donc : $A = \int_1^{\sqrt{e}} f(x)\,dx - \int_{\sqrt{e}}^{e} f(x)\,dx$

Soit F une primitive de f sur $[1, e]$.

Alors : $A = [F(x)]_1^{\sqrt{e}} - [F(x)]_{\sqrt{e}}^{e}$

$= F(\sqrt{e}) - F(1) - F(e) + F(\sqrt{e})$

$= 2F(\sqrt{e}) - F(1) - F(e)$

$\forall x \in]0, +\infty[, \quad F(x) = \int (1 - 2\ln x)\,dx$

$= x - 2\int \ln x\,dx \qquad$ (ipp.)

$= x - 2(x\ln x - x)$

$= 3x - 2x\ln x$

$A = 2\left[3\sqrt{e} - 2\sqrt{e}\cdot\frac{1}{2}\right] - (3-0) - (3e$

$A = (4\sqrt{e} - 3 - e)$ u.a.

$\approx 0{,}88$ u.a.