Ex 2002

1) $\int_0^{\frac{\pi}{6}} \frac{2x+1}{4x^2+1}\,dx = \frac{1}{4}\left[\ln\left(1+\frac{\pi^2}{9}\right)+2\,\mathrm{Arctan}\,\frac{\pi}{3}\right] \cong 0{,}59$

$F(x) = \frac{1}{4}\ln(4x^2+1) + \frac{1}{2}\,\mathrm{Arctan}\,2x$

2) $\int_{\frac{1}{2}}^{1} \frac{x\ln x}{(x^2+1)^2}\,dx = \frac{1}{4}\left[\frac{2x^2\ln x - (x^2+1)\ln(x^2+1)}{x^2+1}\right]_{\frac{1}{2}}^{1}$

$= \frac{1}{4}\left[\ln 5 - \frac{13}{5}\ln 2\right] \cong 0{,}048186$

3) $\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx = \left[x\sqrt{2x-1} - \frac{1}{3}\sqrt{(2x-1)^3}\right]_1^5 = \frac{16}{3}$

4) $\int_0^{\frac{\pi}{6}} \sin^2 2x\cos x\,dx = 4\left[\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x\right]_0^{\frac{\pi}{6}} = \frac{17}{120}$

5) $\lim_{x\to 0} \frac{\mathrm{Arc}\sin 2x - x}{e^{2x} - e^{-x}} \overset{HB}{=} \lim_{x\to 0} \frac{\frac{2}{\sqrt{1-4x^2}} - 1}{2e^{2x} + e^{-x}} = \frac{1}{3}$

6) $f(x) = \frac{x^2-1}{4} - 2\ln x$ $\quad$ $\mathrm{Dom}(f) =]0;+\infty[$.

$f'(x) = \frac{x}{2} - \frac{2}{x} = \frac{(x^2-4)}{2x}$ $\quad$ $f''(x) = \frac{1}{2} + \frac{2}{x^2} > 0\,!$ Arc Convesse sur Dom(f)

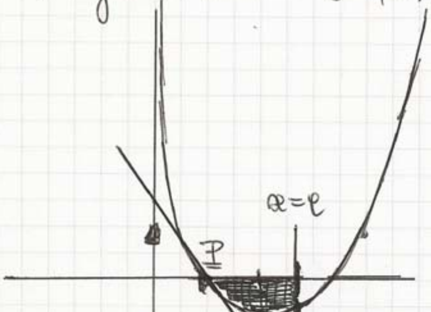
$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty}\left[\frac{x}{4} - \frac{1}{4x} - 2\frac{\ln x}{x}\right] = +\infty$ $\quad$ BP V(+∞).

($\frac{x}{4} \to +\infty$, $\frac{1}{4x} \to 0$, $\frac{\ln x}{x} \to 0$ (HB))

Tangente en P(1;0) : $y = -\frac{3}{2}(x-1)$

x	0		2	
$f'(x)$	‖	−	0	+
$f(x)$	‖ +∞	↘	MIN	↗ +∞

$f(2) \cong -0{,}63$



$\text{Aire} = \int_1^e \ominus f(x)\,dx$

$= \left[2(x\ln x - x) - \frac{x^3}{12} + \frac{x}{4}\right]_1^e$

$= \frac{1}{12}\left[23 + 3e - e^3\right] \cong 0{,}8391$