I)

$f(x) = (x^2 - 3x + 1)e^{x}$

1) $\mathrm{dom}(f) = \mathbb{R}$;

$\lim\limits_{x \to +\infty} f(x) = +\infty$ et $\lim\limits_{+\infty} \frac{f(x)}{x} = +\infty$ (BPV)

$\lim\limits_{x \to -\infty} (e^{x} \cdot x^2)\underbrace{\left(1 - \frac{3}{x} + \frac{1}{x^2}\right)}_{\to 1} = \underline{\underline{0}}$

$\lim\limits_{x \to -\infty} e^{x} \cdot x^2 = \lim\limits_{-\infty} \frac{x^2}{e^{-x}} \underset{(HB)}{=} \lim\limits_{-\infty} \frac{-2x}{-e^{-x}} = \lim\limits_{-\infty} \frac{+2}{e^{-x}} = 0$

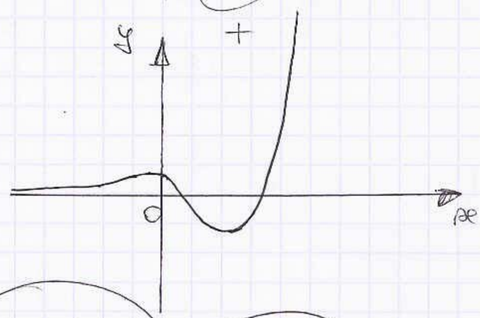
$(AH) \equiv y = 0$.

(racines) $f(x) = 0 \Leftrightarrow x^2 - 3x + 1 = 0$

$\Leftrightarrow x \in \left\{ \frac{3+\sqrt{5}}{2} ; \frac{3-\sqrt{5}}{2} \right\}$.

2) $f'(x) = e^{x}(x^2 - 3x + 1 + 2x - 3) = e^{x}(x^2 - x - 2) = \underbrace{e^{x}}_{+}(x-2)(x+1)$

x	$-\infty$		-1		2		$+\infty$
$f'(x)$		$+$	0	$-$	0	$+$	
$\mathrm{Var}(f)$	0	$\nearrow$	$f(-1) = \frac{5}{e}$	$\searrow$	$f(2) = -e^2$	$\nearrow$	$+\infty$



$\text{Aire} = \int_{-1}^{0} (+)(x^2 - 3x + 1)e^{x}\,dx$ (Méthode) (DIPP)

$= \left[e^{x}(x^2 - 5x + 6)\right]_{-1}^{0} = [6] - \left[\frac{1}{e}(12)\right] = 6\frac{e-2}{e}$

$\cong 1{,}58$ uA

III $f(x)=\dfrac{1}{x\ln x}$ 1) $\mathrm{dom}(f)=]0;1[\,\cup\,]1;+\infty[$.

$\lim\limits_{x\to0^+} f(x)=\lim\limits_{x\to0^+}\dfrac{1}{x\ln x}=-\infty$ (AV)

1) $\lim\limits_{x\to0^+} x\ln x=\lim\limits_{x\to0^+}\dfrac{\ln x}{\frac{1}{x}}=\lim\limits_{x\to0^+}\dfrac{\frac{1}{x}}{\frac{-1}{x^2}}=0^-$

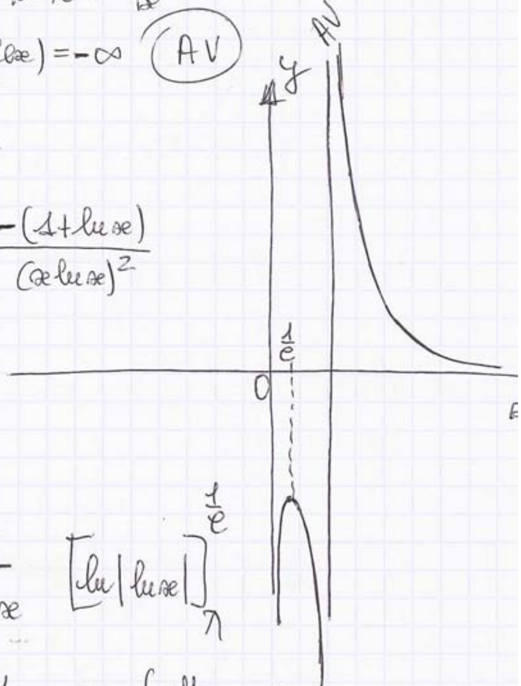
$\lim\limits_{x\to1^+} f(x)=+\infty$ et $\lim\limits_{x\to1^-} f(x)=-\infty$ (AV)

$\lim\limits_{x\to+\infty}\dfrac{1}{x\ln x}=0$ (AH)

2) $f'(x)=\dfrac{-\left[x\cdot\frac{1}{x}+\ln x\right]}{(x\ln x)^2}=\dfrac{-(1+\ln x)}{(x\ln x)^2}$

x	0		$\frac{1}{e}$		1		$+\infty$
$f'(x)$	||	$+$	0	$-$	||	$-$	$-$
Var(f)	|| $-\infty$	↗	MAX	↘ $-\infty$	|| $+\infty$	↘	0^+

$f\left(\frac{1}{e}\right)=-e$



4) $\mathrm{Aire}_\lambda=\int_\lambda^{\frac{1}{e}}(-)\dfrac{dx}{x\ln x}=-\int_\lambda^{\frac{1}{e}}\dfrac{\frac{1}{x}}{\ln x}$ $\quad\left[\ln|\ln x|\right]_\lambda^{\frac{1}{e}}$

CHV $t=\ln x$
$dt=\frac{1}{x}dx$ $\quad\int\dfrac{1}{x\ln x}\,dx=\int\dfrac{dt}{t}=\ln|t|$

$\mathrm{Aire}_\lambda=-\underbrace{\left[\ln\left|\ln\tfrac{1}{e}\right|\right]}_{=0}+\left[\ln|\ln\lambda|\right]=+\ln|\ln\lambda|$

$\mathrm{Aire}_\lambda=1 \Leftrightarrow +\ln|\ln\lambda|=1 \Leftrightarrow \ln|\ln\lambda|=+1$

$\Leftrightarrow |\ln\lambda|=e \Leftrightarrow \cancel{\ln\lambda=e}$ ou $\ln\lambda=-e$

$\Leftrightarrow \lambda=e^{-e}=\dfrac{1}{e^e}$ $\quad$ Car $\lambda<1 \Rightarrow \ln\lambda<0$

$\Leftrightarrow \lambda\simeq 0{,}06598$

IV A) $f(x) = \text{Arc}\sin(1-x)$

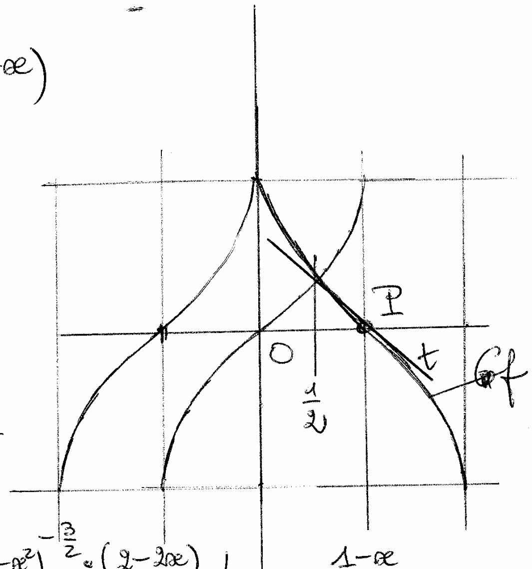
$g(x) = \text{Arc}\sin x$

$\downarrow$

$h(x) = \text{Arc}\sin(x+1)$

$\downarrow$

$f(x) = \text{Arc}\sin(1-x)$



1) $\text{dom}(f) = [0;2]$ $\quad \text{dom}_D(f) =]0;2[$

$f'(x) = \dfrac{-1}{\sqrt{2x-x^2}} = -(2x-x^2)^{-\frac{1}{2}}$

$f''(x) = \frac{1}{2}(2x-x^2)^{-\frac{3}{2}} \cdot \underline{(2-2x)}\ ! = \dfrac{1-x}{\sqrt{(2x-x^2)^3}}$

PT i $= P(1;0)$ Equ. tangente $y = 1-x$

$$I = \int_{\frac{1}{2}}^{1} \text{Arc}\sin(1-x)\,dx = \left[(x-1)\,\text{Arc}\sin(1-x) - \sqrt{2x-x^2}\right]_{\frac{1}{2}}^{1} = \frac{\pi+6\sqrt{3}-12}{12}$$

$u = \text{Arc}\sin(1-x) \qquad u' = \dfrac{-1}{\sqrt{2x-x^2}}$

$v' = 1 \qquad v = x-1$

$\simeq 0{,}1278$

$$\int \text{Arc}\sin(1-x)\,dx = (x-1)\,\text{Arc}\sin(1-x) - \frac{1}{2}\int \frac{2-2x}{\sqrt{2x-x^2}}\,dx$$

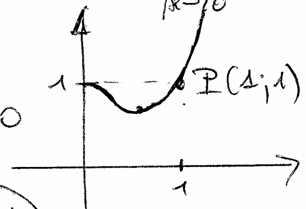
B) $f(x) = x^{x^2} = e^{x^2 \ln x}$ $\qquad \text{dom}(f) =]0;+\infty[= \text{dom}_D(f)$

$f'(x) = x^{x^2}(2x\ln x + x) = x^{x^2+1}(2\ln x + 1)$

x	0	$\frac{1}{\sqrt{e}}$	$+\infty$
	$\searrow$		$\nearrow +\infty$

On peut prolonger par continuité en $x=0$ car $\begin{cases} \lim\limits_{x\to 0} f(x) = 1. \\ \text{et } \lim\limits_{x\to 0^+} \dfrac{f(x)-1}{x} = \lim\limits_{x\to 0^+} f'(x) = 0 \end{cases}$

TANGENTE en P $\equiv$ $y = x$



C) $t = 3^x$ $\quad t^2 - t - 6 < 0 \iff (t-3)(t+2) < 0$

$\iff (3^x-3)(3^x+2) < 0$

$\iff \cancel{-2<}\ 3^x < 3 \iff x < 1 \Rightarrow S =]-\infty;1[.$